[a,1] [b]

[a]

[b]

λ

Λ^I

Λ^I
Λ^I

λ

λ
Λ^I K
λ

λ

---

[1]

λ Λ^K Λ^I

Λ^I

$$(M\! - >_\beta N \quad \& \quad M \in \infty_\beta \quad) \Longrightarrow \quad N \in \infty_\beta \ ,$$

Λ^I M N

$$M \in \mathrm{WN}_\beta \quad \Longrightarrow \quad M \in \mathrm{SN}_\beta \ ,$$

Λ^I M

M

$M \in \mathrm{WN}_\beta \cap \mathrm{SN}_\beta = \mathrm{SN}_\beta$ $M \notin \infty_\beta$

$M \notin \mathrm{WN}_\beta \cup \mathrm{SN}_\beta = \mathrm{WN}_\beta$ $M \in \infty_\beta$

$M\! - >_\beta N$ $N \in \infty_\beta$ $M \in \mathrm{WN}_\beta$

Λ^I β

M M

$$\begin{array}{ccccccccc} M_0 & \xrightarrow{->_\beta} & M_1 & \xrightarrow{->_\beta} & M_2 & \xrightarrow{->_\beta} & \cdots & \xrightarrow{->_\beta} & M_n \in \mathrm{NF}_\beta \\ \downarrow{\scriptstyle ->_\beta} & & \downarrow{\scriptstyle ->_\beta} & & \downarrow{\scriptstyle ->_\beta} & & & & \downarrow{\scriptstyle ->_\beta} \end{array}$$

$M_1, \ldots, M_n$ Λ^I Λ^I β

λ λ

K

λ

λ λ

Λ^K

[2]

Λ^I

λ

β λ

λ

λ

λ

λ

λ

λ

2

Λ^I

Λ^I

λ

Λ^K

Λ^K λ α
$I \equiv \lambda x.x$ $\omega \equiv \lambda x.x\,x$
$\Omega \equiv \omega\,\omega$ $\equiv$ α

$M \subseteq N$ M N $\mathrm{FV}(M)$
M Λ^I λ
$\lambda x.P$ $x \in \mathrm{FV}(P)$ C
$\square$ $C[M]$ C
M M

θ
$\operatorname{dom}\theta$ θ V
$\theta_{|V}$ V θ
$\theta_{\setminus V}$ $\theta_{|x}$ $\theta_{\setminus x}$ $V = \{x\}$ $\theta\phi$
θ ϕ σ

$$\sigma = \{x := P\phi \mid x := P \in \theta\} \cup \{x := P \mid x := P \in \phi_{\setminus \operatorname{dom}\theta}\}$$

M θ M

$$M \subseteq \theta \iff M \subseteq N \qquad x := N \in \theta$$

$M, N \in \Lambda^K$

θ $N \subseteq M\theta$ $N \not\subseteq \theta$

$N \equiv \lambda x.P$ $N' \equiv \lambda y.P' \subseteq M$ $N \equiv N'\theta$

$N \equiv P\,Q$ $N' \equiv P'\,Q' \subseteq M$ $N \equiv N'\theta$

$\mathbf{R}$ L L

$\mathbf{R}$ $- >_{\mathbf{R}}$

$\twoheadrightarrow_{\mathbf{R}}$ $\twoheadrightarrow^{+}_{\mathbf{R}}$

$=_{\mathbf{R}}$

$\twoheadrightarrow_{\mathbf{R}}$

L

$\mathbf{R}$ L

$$M_0 - >_{\mathbf{R}} M_1 - >_{\mathbf{R}} \cdots$$

$\mathbf{R}$ M_0 M_0 $\mathbf{R}$

M_n

n

$$\begin{aligned}
\infty_{\mathbf{R}} &= \{M \in L \mid M \quad \mathbf{R} \quad \} \\
\mathrm{NF}_{\mathbf{R}} &= \{M \in L \mid M \quad \mathbf{R} \quad \} \\
\mathrm{SN}_{\mathbf{R}} &= \{M \in L \mid M \quad \mathbf{R} \quad \} \\
\mathrm{WN}_{\mathbf{R}} &= \{M \in L \mid M \quad \mathbf{R} \quad N \in \mathrm{NF}_{\mathbf{R}} \} \\
\mathrm{UN}_{\mathbf{R}} &= \{M \in L \mid M \in \mathrm{WN}_{\mathbf{R}} \Rightarrow M \in \mathrm{SN}_{\mathbf{R}}\} \\
\mathrm{CON}_{\mathbf{R}} &= \{M \in L \mid \forall N : M - >_{\mathbf{R}} N \;\&\; M \in \infty_{\mathbf{R}} \Rightarrow N \in \infty_{\mathbf{R}}\}
\end{aligned}$$

$\infty_{\mathbf{R}}$ $\mathbf{R}$ $\mathrm{NF}_{\mathbf{R}}$

$\mathbf{R}$ $\mathrm{SN}_{\mathbf{R}}$

$\mathbf{R}$ $\mathrm{WN}_{\mathbf{R}}$

$\mathbf{R}$ $\mathrm{UN}_{\mathbf{R}}$ $\mathbf{R}$

$\mathrm{CON}_{\mathbf{R}}$

$\mathbf{R}$

$\mathbf{R}$ $F : L \to L$ $M -\!\!> _{\mathbf{R}} F(M)$
$M \in L \setminus \textsc{nf}_{\mathbf{R}}$ $F(M) \equiv M$ $M \in \textsc{nf}_{\mathbf{R}}$

$\textsc{un}_{\mathbf{R}}$
$M \in \textsc{sn}_{\mathbf{R}}$
$M \notin \infty_{\mathbf{R}}$ Λ^K
β

$$(\lambda x.P)\, Q \quad \beta_\lambda \quad P\{x := Q\} \;.$$

$\lambda x.P$ $(\lambda x.P)\, Q$
$\lambda x.P$ β_λ $(\lambda x.P)\, Q$ I I
$x \in \textsc{fv}(P)$ K K
$x \notin \textsc{fv}(P)$ $\mathbf{K}\, P$ $\mathbf{K}\, P\, Q$
λ β_λ

$\mathbf{R}$ L

$\textsc{un}_{\mathbf{R}} \subseteq \textsc{con}_{\mathbf{R}}$
L' L L' $\mathbf{R}$ $L' \subseteq \textsc{con}_{\mathbf{R}}$
$L' \subseteq \textsc{un}_{\mathbf{R}}$

$\Box$
$\to$ $\equiv$

Δ Δ Δ' C
C

$$(C[\Delta] -\!\!>_\beta C[\Delta'] \;\;\&\;\; C[\Delta] \in \infty_\beta) \Longrightarrow C[\Delta'] \in \infty_\beta \;.$$

$\Delta \equiv I\, I$ $\Delta \equiv \mathbf{K}\, \Omega\, I$ $\Delta \equiv \mathbf{K}\, I\, \Omega$

$x\,((\lambda y.z)\,\Omega)\,\Omega$

$(\lambda y.z)\,\Omega$

$C \equiv []$ $C[(\lambda y.z)\,\Omega] \in \infty_\beta$ $C[z] \notin \infty_\beta$

I Λ^K

C Λ^K Δ I Δ'

$$C[\Delta] \in \infty_\beta \quad \Longrightarrow \quad C[\Delta'] \in \infty_\beta \ .$$

Λ^K Λ^K I K
$\mathbf{K}\,A\,B$ A B

Λ^K
λ

I
K K $\mathbf{K}\,A\,B$ B

K

$\mathbf{K}\,A\,B$

B C

B A

A B

B A

θ A B

$P, Q \in \Lambda^K$ P

Q $P \succeq_{\infty,\beta} Q$ θ

$$Q\theta \in \infty_\beta \Longrightarrow P\theta \in \infty_\beta \ .$$

K $\mathbf{K}\,A\,B$ $A \succeq_{\infty,\beta} B$

$P\theta \in \infty_\beta$ P

θ

$\mathbf{K}\,A\,B$

$\mathbf{K}\,A\,B$

$\succeq_{\infty,\beta}$ $\succeq^{\text{SN}}_{\infty,\beta}$

θ SN

$\mathbf{K}\,A\,B$

$A \succeq^{\text{SN}}_{\infty,\beta} B$

$$\mathbf{K}\,A\,B \equiv \mathbf{K}\,x\,y.$$

SN θ $A\theta \in \infty_\beta \Leftarrow B\theta \in \infty_\beta$ y

SN $B\theta \notin \infty_\beta$

$A \not\succeq_{\infty,\beta} B$ $\theta = \{y := \Omega\}$

K $\mathbf{K}\,A\,B$ $\textsc{fv}(A) = \textsc{fv}(B)$

SN

$\theta = \{x := u\,u\}$ $\theta' = \{u := \omega\}$

SN $\theta\,\theta' = \{u := \omega, x := \omega\,\omega\}$

$\Omega \in \infty_\beta$

K

$\Delta \equiv \mathbf{K}\, A\, B$

C $C[\Delta]$

Δ λ

$\underline{\Lambda}^K$ $\underline{\lambda}$ λ

$$
\begin{aligned}
& x \in \underline{\Lambda}^K && x \\
& P \in \underline{\Lambda}^K && \Longrightarrow \lambda x.P \in \underline{\Lambda}^K \\
& P, Q \in \underline{\Lambda}^K && \Longrightarrow P\, Q \in \underline{\Lambda}^K \\
& P, Q \in \underline{\Lambda}^K && \Longrightarrow (\underline{\lambda} x.P)\, Q \in \underline{\Lambda}^K
\end{aligned}
$$

$(\underline{\lambda} x.P)\, Q$

λ^* $\lambda^* ::= \lambda \mid \underline{\lambda}$

$\underline{\beta}$ β $\underline{\Lambda}^K$

$$(\underline{\lambda} x.P)\, Q \;\underline{\beta}\; P\{x := Q\}$$

$$(\lambda x.P)\, Q \;\beta\; P\{x := Q\}$$

$P, Q \in \underline{\Lambda}^K$

β^* $\beta^* = \underline{\beta} \cup \beta$

$\underline{\Lambda}^K$ Λ^K β $\underline{\Lambda}^K$

β^*

$\underline{\Lambda}^K$ Λ^K

φ

$\underline{\Lambda}^K$ M

$\varphi(M) \in \Lambda^K$ M

$$
\begin{aligned}
\varphi(x) &\equiv x \\
\varphi(\lambda x.P) &\equiv \lambda x.\varphi(P) \\
\varphi(P\, Q) &\equiv \varphi(P)\, \varphi(Q) \\
\varphi((\underline{\lambda} x.P)\, Q) &\equiv \varphi(P)\{x := \varphi(Q)\}
\end{aligned}
$$

$\underline{\beta}$

$\underline{\beta}$

$M, N \in \underline{\Lambda}^K$

$$\varphi(M\{x := N\}) \equiv \varphi(M)\{x := \varphi(N)\}$$

S ∞_{β^*} F^* β^*

S F^* $L \in S$ $F^*(L) \in S$

$L \in S$ $L- >_\beta F^*(L)$ $\varphi(L) \twoheadrightarrow^+_\beta \varphi(F^*(L))$

M Λ^K $M \equiv C[(\lambda x.P)\, Q]$

C Λ^K P Q

$M \in S$ $C[P\{x := Q\}]$

S

F^*

M

S

S

S F^*

$\underline{\Lambda}^{\underline{K}}_{\text{good}}$

K

β^* F^*

F^* $S = \infty_{\beta^*} \cap \underline{\Lambda}^{\underline{K}}_{\text{good}}$

$\underline{\Lambda}^{\underline{K}}_{\text{good}}$

$\underline{\Lambda}^{\underline{K}}$ $\underline{\Lambda}^K$ $(\underline{\lambda} x.P)\, Q \subseteq$

M $x \notin \textsc{fv}(P)$ $M \in \underline{\Lambda}^{\underline{K}}$

$\underline{\Lambda}^K$ $\underline{\Lambda}^{\underline{K}}$ K

K

$\underline{\Lambda}^{\underline{K}}$ $\underline{\Lambda}^{\underline{K}}$

$M \in \underline{\Lambda}^K$ $\underline{\mathbf{K}}\ A\ B \subseteq M$
$\underline{\mathbf{K}}\ A\ B$ $\underline{\Lambda}^K_{\text{good}}$ $\underline{\Lambda}^K$ $\{M \in \underline{\Lambda}^K \mid M \quad\}$

$\underline{\Lambda}^K_{\text{good}}$ β^*

M $\underline{\Lambda}^K_{\text{good}}$ N $\underline{\Lambda}^K$ $M -\!>_{\beta^*} N$
$N \in \underline{\Lambda}^K_{\text{good}}$

$(\underline{\lambda} z.A)\ B \subseteq N$

$z \notin \text{FV}(A)$ $A \succeq_{\infty,\beta^*} B$. $*$

$(\underline{\lambda} z.A)\ B \subseteq M$
$M -\!>_{\beta^*} N$ $*$

$(\lambda^* x.P)\ Q -\!>_{\beta^*} P\{x := Q\}$
$(\underline{\lambda} z.A)\ B \subseteq Q$ $*$ $(\underline{\lambda} z.A)\ B \subseteq Q \subseteq M$

$(\underline{\lambda} z.A')\ B' \subseteq P$ $(\underline{\lambda} z.A)\ B \equiv ((\underline{\lambda} z.A')\ B')\{x := Q\}$
$(\underline{\lambda} z.A')\ B' \subseteq P \subseteq M$ $z \notin A'$ $z \notin A$
$A'\phi \in \infty_{\beta^*} \Leftarrow B'\phi \in \infty_{\beta^*}$
ϕ θ
$\phi = \{x := Q\}\theta$

$$B\theta \equiv B'\phi \in \infty_{\beta^*} \Longrightarrow A'\phi \in \infty_{\beta^*} \equiv A\theta \ .$$

$x\ B \subseteq P$ $Q \equiv \underline{\lambda} x.A$
$M \in \underline{\Lambda}^K_{\text{good}}$ Q

$\lambda^* x.P -\!>_{\beta^*} \lambda^* x.P'$ $P -\!>_{\beta^*} P'$ $(\underline{\lambda} z.A)\ B \subseteq \lambda^* x.P'$
$(\underline{\lambda} z.A)\ B \subseteq P'$
$*$ $(\underline{\lambda} z.A)\ B$
$P\ Q -\!>_{\beta^*} P'\ Q$ $Q\ P -\!>_{\beta^*} Q\ P'$ $P -\!>_{\beta^*} P'$

$\underline{\lambda} x.A$
$(\underline{\lambda} z.A)\ B \subseteq P'\ Q$ $(\underline{\lambda} z.A)\ B \subseteq P'$ $(\underline{\lambda} z.A)\ B \subseteq Q$
$*$
$*$ $Q \subseteq M$

$\underline{\Lambda}^K_{\text{good}}$
$-\!>_{\beta^*}$ $\square$

F_2^*

$$F^* : \infty_{\beta^*} \to \underline{\Lambda}^K$$

$$\begin{array}{lll} F^*(x\,P_1\,\ldots\,P_m\,\ldots\,P_n) & \equiv x\,P_1\,\ldots\,P_{m-1}\,F^*(P_m)\,P_{m+1}\,\ldots\,P_n \\ & \quad P_m \in \infty_{\beta^*} \qquad P_i \in \mathrm{SN}_{\beta^*} \qquad i < m \\ F^*(\lambda x.P) & \equiv \lambda x.F^*(P) \\ F^*((\lambda^* x.P_0)\,P_1\,\ldots P_n) & \equiv P_0\{x := P_1\}\,P_2\,\ldots\,P_n \\ & \quad P_0, P_1 \in \mathrm{SN}_{\beta^*} \\ F^*((\lambda^* x.P_0)\,P_1\,\ldots\,P_n) & \equiv (\lambda^* x.P_0)\,F^*(P_1)\,P_2\,\ldots\,P_n \\ & \quad P_0 \in \mathrm{SN}_{\beta^*} \qquad P_1 \in \infty_{\beta^*} \\ F^*((\lambda^* x.P_0)\,P_1\,\ldots\,P_n) & \equiv (\lambda^* x.F^*(P_0))\,P_1\,\ldots\,P_n \\ & \quad P_0 \in \infty_{\beta^*} \end{array}$$

$n \geq 1$ $m \leq n$

F^*

$\underline{\Lambda}^K$

F^*

β^* $\underline{\Lambda}^K$ F^*

M $\underline{\Lambda}^K$ $M -\!>_{\beta^*} F^*(M)$ $F^*(M)$

M $F^*(M)$

□

$\underline{\Lambda}^K_{\text{good}} \cap \infty_{\beta^*}$ $\underline{\Lambda}^K$

F^*

M $\underline{\Lambda}^K_{\text{good}} \cap \infty_{\beta^*}$

$$M -\!>_{\beta} F^*(M) \implies \varphi(M) \twoheadrightarrow^+_{\beta} \varphi(F^*(M)) \ .$$

$$M -\!>_{\beta} F^*(M) \ . \qquad *$$

M

$M \equiv x\,P_1\,\ldots\,P_m\,\ldots\,P_n$ $n \geq 1$ $P_m \in \infty_{\beta^*}$ $P_i \in \mathrm{SN}_{\beta^*}$
$i < m \leq n$

$M \equiv \lambda x.P$ $n \geq 1$
$M \equiv (\lambda x.P_0)\, P_1\, \ldots\, P_n$

$$\varphi(M) \equiv (\lambda x.\varphi(P_0))\, \varphi(P_1)\, \ldots\, \varphi(P_n)\ .$$

$P_0, P_1 \in \mathrm{SN}_{\beta^*}$

$P_0, P_1 \in \mathrm{SN}_{\beta^*}$

$$F^*(M) \equiv P_0\{x := P_1\}\, P_2\, \ldots\, P_n$$

$$\varphi(F^*(M)) \equiv \varphi(P_0\{x := P_1\})\, \varphi(P_2)\, \ldots\, \varphi(P_n)\ .$$

$$\begin{aligned}\varphi(M) &\equiv (\lambda x.\varphi(P_0))\, \varphi(P_1)\, \varphi(P_2)\, \ldots\, \varphi(P_n)\\ &- >_\beta \varphi(P_0)\{x := \varphi(P_1)\}\, \varphi(P_2)\, \ldots\, \varphi(P_n)\\ &= \varphi(P_0\{x := P_1\})\, \varphi(P_2)\, \ldots\, \varphi(P_n)\\ &\equiv \varphi(F^*(M))\end{aligned}$$

$P_0 \in \infty_{\beta^*}$ $P_0 \in \mathrm{SN}_{\beta^*}$ $P_1 \in \infty_{\beta^*}$

$M \equiv (\underline{\lambda} x.P_0)\, P_1\, \ldots\, P_n$ $n \geq 1$

$$\varphi(M) \equiv \varphi(P_0)\{x := \varphi(P_1)\}\, \varphi(P_2)\, \ldots\, \varphi(P_n)\ .$$

$P_0, P_1 \in \mathrm{SN}_{\beta^*}$ $*$

$$M \equiv (\underline{\lambda} x.P_0)\, P_1\, \ldots\, P_n - >_\beta P_0\{x := P_1\}\, P_2\, \ldots\, P_n \equiv F^*(M)\ ,$$

$(\underline{\lambda} x.P_0)\, P_1 - >_\beta P_0\{x := P_1\}$
β

$P_0 \in \mathrm{SN}_{\beta^*}$ $P_1 \in \infty_{\beta^*}$ $M \in \underline{\Lambda}^{K}_{\mathrm{good}}$
$M \in \underline{\Lambda}^{K}_{\mathrm{good}}$
$(\underline{\lambda} x.P_0)\, P_1$ K $P_0 \succeq_{\infty,\beta^*} P_1$
$\emptyset$ $P_0\emptyset \in$
SN_{β^*} $P_1\emptyset \in \infty_{\beta^*}$
$P_0 \in \infty_{\beta^*}$

$$F^*(M) \equiv (\underline{\lambda} x.F^*(P_0))\, P_1\, \ldots\, P_n$$

$$\varphi(F^*(M)) \equiv \varphi(F^*(P_0))\{x := \varphi(P_1)\}\, \varphi(P_2)\, \ldots\, \varphi(P_n)\ .$$

$*$ $P_0 - >_\beta F^*(P_0)$

$\varphi(P_0) \twoheadrightarrow_\beta^+ \varphi(F^*(P_0))$

$$\begin{aligned}\varphi(M) &\equiv \varphi(P_0)\{x := \varphi(P_1)\}\, \varphi(P_2)\ \ldots\ \varphi(P_n)\\ &\twoheadrightarrow_\beta^+ \varphi(F^*(P_0))\{x := \varphi(P_1)\}\, \varphi(P_2)\ \ldots\ \varphi(P_n)\\ &\equiv \varphi(F^*(M))\end{aligned}$$

$\varphi(P_0) \twoheadrightarrow_\beta^+ \varphi(F^*(P_0))$

$\varphi(P_0)\theta \twoheadrightarrow_\beta^+ \varphi(F^*(P_0))\theta$ θ

$$M - >_\beta F^*(M) \Longrightarrow \varphi(M) \twoheadrightarrow_\beta^+ \varphi(F^*(M))\ .$$

□

$\mathbf{K}\, A\, B$

C $C[\mathbf{K}\, A\, B] \in \infty_\beta$ $C[A] \in \infty_\beta$

C $C[\mathbf{K}\, A\, B] \in \infty_\beta$

$S = \underline{\Lambda}^{K}_{\text{good}} \cap \infty_{\beta^*}$

□

Λ^K

M $M \in \text{WN}_\beta$

$M \in \text{SN}_\beta$

Λ^K Λ^I Λ^I

β

β

β

β

Λ^K

β

$$(\lambda y.\mathbf{K}\, x\, y)\, \Omega \quad - >_\beta \quad \mathbf{K}\, x\, \Omega\ .$$

$\lambda y.\mathbf{K}\,x\,y$ Ω I

SN y $\mathbf{K}\,x\,y$

$\mathbf{K}\,x\,\Omega$ $\mathbf{K}\,x\,\Omega \in \infty_\beta$

$x \notin \infty_\beta$

λ

Λ^K

λ

K

K

$$((\lambda x.\mathbf{K}\,x)\,I)\,\Omega - >_\beta \mathbf{K}\,I\,\Omega \;.$$

I

K

K

K K $\mathbf{K}\,x$

K

Λ^K K

K

K

$(\lambda x.P)\,Q$

M N Λ^K $M - >_\beta N$ M

$M \in \infty_\beta$ $N \in \infty_\beta$

C $M \equiv C[(\lambda x.P)\,Q] - >_\beta C[P\{x := Q\}] \equiv$

N $(\lambda x.P)\,Q$ I

$(\lambda x.P)\,Q$ K

□

K

λx λy

$$\lambda x.x\,(\lambda y.\mathbf{K}\;I\;I)\;(I\;I)\ .$$

K

K

$\Lambda^{\text{exp}}_{\text{full}}$ K Λ^{K}

$$
\begin{aligned}
& x \in \Lambda^{\text{exp}}_{\text{full}} \\
& P \in \Lambda^{\text{exp}}_{\text{full}}\ \&\ x \in \textsc{fv}(P) \Longrightarrow \lambda x.P \in \Lambda^{\text{exp}}_{\text{full}} \\
& P, Q \in \Lambda^{\text{exp}}_{\text{full}} \Longrightarrow \mathbf{K}\;P\;Q \in \Lambda^{\text{exp}}_{\text{full}} \\
& P, Q \in \Lambda^{\text{exp}}_{\text{full}} \Longrightarrow P\;Q \in \Lambda^{\text{exp}}_{\text{full}}
\end{aligned}
$$

x

Λ^{exp} K K

Λ^{K}

$$
\begin{aligned}
& P_1, \ldots, P_n \in \Lambda^{\text{exp}} \Longrightarrow x\;P_1\ \ldots\ P_n \in \Lambda^{\text{exp}} \\
& P \in \Lambda^{\text{exp}} \Longrightarrow \lambda x.P \in \Lambda^{\text{exp}} \\
& P, Q, R_1, \ldots, R_n \in \Lambda^{\text{exp}}_{\text{full}} \Longrightarrow (\lambda x.P)\;Q\;R_1 \ldots\ R_n \in \Lambda^{\text{exp}}
\end{aligned}
$$

x $n \geq 0$

$\mathbf{K}\,(\mathbf{K}\,(\lambda x.x)\,(\lambda y.y))$ K K

$\mathbf{K}\,(\mathbf{K}\,(\lambda x.x))\,(\lambda y.y)$

K K

K

K K

$(\lambda x.P)\,Q\,R$ $P\{x := Q\}\,R$

P Q

R R

$M \in \Lambda^K$ K $M \in \text{UN}_\beta$

K

$$(\lambda y.\mathbf{K}\,x\,y)\,\Omega \quad ->_\beta \quad \mathbf{K}\,x\,\Omega\ .$$

K

SN K

Λ^K β

$$\begin{array}{ccc}
\Lambda^{\text{exp}}_{\text{good}} \ni M & \xrightarrow{->_\beta} & \cdots \\
\downarrow{\scriptstyle ->_\beta} & & \\
\Lambda^{\text{exp}}_{\text{good}} \ni F(M) & \xrightarrow{->_\beta} & \cdots \\
\downarrow{\scriptstyle ->_\beta} & & \\
\Lambda^{\text{exp}}_{\text{good}} \ni F^2(M) & \xrightarrow[->_\beta]{} & \cdots \\
\downarrow{\scriptstyle ->_\beta} & & \\
 & & \\
\downarrow{\scriptstyle ->_\beta} & & \\
\text{NF}_\beta \cap \Lambda^{\text{exp}}_{\text{good}} \ni F^n(M) & \xrightarrow[->_\beta]{} & \cdots
\end{array}$$

$\Lambda^{\text{exp}}_{\text{good}}$ K F

$M \in \text{WN}_\beta$ F M

$M \in$

$\Lambda^{\text{exp}}_{\text{good}}$ F

$\Lambda^{\text{exp}}_{\text{good}} \subseteq \Lambda^K$ F

$M \in \text{WN}_\beta$ $M \in \infty_\beta$ $F^n(M)$

n $M \in \text{UN}_\beta$

F_l Λ^K

$M \in \Lambda^K \setminus \mathrm{NF}_\beta$ M

$$F_l(x\,P_1\,\ldots\,P_n) \equiv x\,P_1\,\ldots\,P_{m-1}\,F_l(P_m)\,P_{m+1}\,\ldots\,P_n$$

$P_i \in \mathrm{NF}_\beta$ $i < m$ $P_m \notin \mathrm{NF}_\beta$

$$F_l(\lambda x.P) \equiv \lambda x.F_l(P)$$

$$F_l((\lambda x.P_0)\,P_1\,P_2\,\ldots\,P_n) \equiv P_0\{x := P_1\}\,P_2\,\ldots\,P_n.$$

$n \geq 1$ $m \leq n$ $M \in \mathrm{NF}_\beta$ $F_l(M) \equiv M$

$M -\ >_\beta\ F_l(M)$ $-\ >_\beta$

K K

K

M Λ^K $M \in \Lambda^{\exp}_{\text{full}}$ $M \in \Lambda^{\exp}$

$M \in \Lambda^{\exp}_{\text{full}}$ K

$M \equiv x\,Q_1\,\ldots\,Q_n$

$M \equiv (\lambda x.P)\,Q_1\,\ldots\,Q_n$

$n \geq 0$ $Q, P_1, \ldots P_n \in \Lambda^{\exp}_{\text{full}}$

M $\square$

K

K

K K

K

M N Λ^K $M, N \in \Lambda^{\exp}_{\text{full}}$

$M\{x := N\} \in \Lambda^{\exp}_{\text{full}}$

M $\square$

K

$\mathbf{K}\,x\,y$ $\mathbf{K}\,z$ K $(\mathbf{K}\,x\,y)\{y := \mathbf{K}\,z\} \equiv$
$\mathbf{K}\,x\,(\mathbf{K}\,z)$
$\Lambda^{\mathrm{exp}}_{\mathrm{good}}$ F_l

M K Λ^K $F_l(M)$ K

K β
$(\lambda z.\lambda x.\mathbf{K}\,z\,x)\,I$[3] K $(\lambda z.\lambda x.z)\,I \equiv (\lambda z.\mathbf{K}\,z)\,I$

$M \in \Lambda^{\mathrm{exp}}$ $F_l(M) \in \Lambda^{\mathrm{exp}}$
$M \in \mathrm{NF}_\beta$ $F_l(M) \equiv M \in \Lambda^{\mathrm{exp}}$
$M \notin \mathrm{NF}_\beta$ M

$M \equiv x\,P_1\,\ldots\,P_n$ $n \geq 1$ $m \leq n$ $P_m \notin \mathrm{NF}_\beta$
$P_i \in \mathrm{NF}_\beta$ $i < m$ $F_l(M) \equiv x\,P_1\,\ldots\,P_{m-1}\,F_l(P_m)\,P_{m+1}\,\ldots\,P_n$
$M \in \Lambda^{\mathrm{exp}}$ $P_j \in \Lambda^{\mathrm{exp}}$ j
$1 \leq j \leq n$ $F_l(P_m) \in \Lambda^{\mathrm{exp}}$
$F_l(M) \in \Lambda^{\mathrm{exp}}$
$M \equiv \lambda x.P$
$M \equiv (\lambda x.P_0)\,P_1\,\ldots\,P_n$ $n \geq 1$ $F_l(M) \equiv P_0\{x := P_1\}\,P_2\,\ldots\,P_n$
$M \in \Lambda^{\mathrm{exp}}$ $P_i \in \Lambda^{\mathrm{exp}}_{\mathrm{full}}$ i
$0 \leq i \leq n$ $P_0\{x := P_1\} \in \Lambda^{\mathrm{exp}}_{\mathrm{full}}$
$F_l(M) \in \Lambda^{\mathrm{exp}}_{\mathrm{full}}$ $F_l(M) \in \Lambda^{\mathrm{exp}}$

K

$\square$

$M \in \Lambda^K$ K $F_l(M)$

$M \in \Lambda^{\mathrm{exp}}_{\mathrm{good}}$ $F_l(M)$
$M \in \mathrm{NF}_\beta$ $F_l(M) \equiv M$
$M \notin \mathrm{NF}_\beta$ M

---

[3] $\Lambda^{\mathrm{exp}}_{\mathrm{good}}$ K
β
$\mathrm{FV}(P) = \mathrm{FV}(Q)$
K $\mathbf{K}\,P\,Q$ β
K

$M \equiv x\, P_1\, \ldots\, P_n$ $n \geq 1$ $m \leq n$ $P_m \notin \mathrm{NF}_\beta$
$P_i \in \mathrm{NF}_\beta$ $i < m$ $F_l(M) \equiv x\, P_1\, \ldots\, P_{m-1}\, F_l(P_m)\, P_{m+1}\, \ldots\, P_n$
K $\Delta \equiv \mathbf{K}\, S\, T \subseteq F_l(M)$
$\Delta \subseteq P_i$ i $i \neq m$ $\Delta \subseteq P_i \subseteq M$ Δ

$\Delta \subseteq F_l(P_m)$ $F_l(P_m)$
Δ
K $F_l(M)$ $F_l(M)$
$M \equiv \lambda x.P$
$M \equiv (\lambda x.P_0)\, P_1\, \ldots\, P_n$ $n \geq 1$ $F_l(M) \equiv P_0\{x := P_1\}\, P_2\, \ldots\, P_n$
$M \in \Lambda^{\mathrm{exp}}$ $P_i \in \Lambda^{\mathrm{exp}}_{\mathrm{full}}$ i
$0 \leq i \leq n$ $\Delta \equiv \mathbf{K}\, S\, T \subseteq F_l(M)$

$\Delta \subseteq P_0\{x := P_1\}$ $\Delta \not\subseteq P_1$ $S \succeq_{\infty,\beta} T$

$\Delta' \equiv \mathbf{K}\, S'\, T' \subseteq P_0$ $\Delta'\{x := P_1\} \equiv \Delta$ M
$S' \succeq_{\infty,\beta} T'$ θ

$$\infty_\beta \ni S\theta \equiv (S'\{x := P_1\})\theta \Longleftarrow (T'\{x := P_1\})\theta \equiv T\theta \in \infty_\beta$$

Δ
$x\, T' \subseteq P_0$ $P_1 \equiv \mathbf{K}\, S$ $T \equiv T'\{x := P_1\}$

$P_1 \equiv \mathbf{K}\, S \in \Lambda^{\mathrm{exp}}_{\mathrm{full}}$
$\Delta \subseteq P_i$ i $i \geq 1$ $\Delta \subseteq M$ Δ
$\Delta \equiv (P_0\{x := P_1\})\, P_2 \equiv (\lambda y.Q)\, P_2$
Δ K Δ K
$\mathbf{K}\, Q\, P_2$ $P_0 \equiv \mathbf{K}\, Q'$ $Q \equiv Q'\{x := P_1\}$ $P_0 \equiv x$
$P_1 \equiv \mathbf{K}\, Q$ $P_0 \in \Lambda^{\mathrm{exp}}_{\mathrm{full}}$ $P_1 \in \Lambda^{\mathrm{exp}}_{\mathrm{full}}$
K $F_l(M)$ $F_l(M)$

K
$\square$

$\mathbf{K}\, P\, Q -\!\!>_\beta \mathbf{K}\, P'\, Q$
$P -\!\!>_\beta P'$ P
K

$$P \succeq_{\infty,\beta} Q \;\&\; P -\!\!>_\beta P' \quad \Longrightarrow \quad P' \succeq_{\infty,\beta} Q\ .$$

K

$M \in \Lambda^K$ K
$M \in \text{WN}_\beta$ $F_l(M)$

$$M -\!\!>_\beta F_l(M) -\!\!>_\beta F_l^2(M) -\!\!>_\beta \cdots -\!\!>_\beta F_l^n(M) \in \text{NF}_\beta$$

$n \in \mathbb{N}$

$M \in \text{SN}_\beta$
$M \in \infty_\beta$ $i \geq 0$
$F_l^i(M)$ K $F_l^n(M) \in \infty_\beta$
$F_l^n(M) \in \text{NF}_\beta$ $M \in \text{SN}_\beta$ □

$\Lambda_{\langle\rangle}$

λ $\langle M, M' \rangle$
$\pi_i(M)$
λ
case

β
$\pi_1(\langle x, \Omega \rangle)$ $(\lambda x.\langle \lambda y.y, z \rangle)\,\Omega$

$$\pi_1(\langle x, \Omega \rangle) -\!\!>_\beta x$$

$$(\lambda x.\langle I, z \rangle)\,\Omega -\!\!>_\beta \langle I, z \rangle$$

Ω β

$\Lambda^{[]}_{\langle\rangle}$ $\Lambda_{\langle\rangle}$

$\iota : \Lambda_{\langle\rangle} \to \Lambda^{[]}_{\langle\rangle}$ $\Lambda_{\langle\rangle}$ M
$\iota(M)$
M $\Lambda_{\langle\rangle}$
$\Lambda^{[]}_{\langle\rangle}$

$\underline{\bullet}$ β

ι $\underline{\bullet}$

$M \in \Lambda_{\langle\rangle}$

$\underline{\iota(M)}$

$\underline{\iota(M)} \in \mathrm{WN}_\beta$ $M \in$

SN_β

$$\underline{\iota(M)} \in \mathrm{WN}_\beta \Longrightarrow \underline{\iota(M)} \in \mathrm{SN}_\beta \Longrightarrow \iota(M) \in \mathrm{SN}_{\beta\pi} \Longrightarrow M \in \mathrm{SN}_\beta$$

π β $\Lambda_{\langle\rangle}$

Λ^I

$\Lambda_{\langle\rangle}$

λ

$\Lambda_{\langle\rangle}$ λ $\langle M, M\rangle$

$\pi_i(M)$

$$\Lambda_{\langle\rangle} \ni M ::= x \mid \lambda x.M \mid M\ M \mid \langle M, M\rangle \mid \pi_1(M) \mid \pi_2(M)$$

$i = 1, 2$ x

$\Lambda_{\langle\rangle}$

$$(\lambda x.P)\ Q\ \ \beta_\lambda\ \ P\{x := Q\}$$

$$\pi_i(\langle P_1, P_2\rangle)\ \beta_{\langle\rangle}\ \ P_i$$

$P, Q, P_1, P_2 \in \Lambda_{\langle\rangle}$

β $\beta = \beta_\lambda \cup \beta_{\langle\rangle}$

β_λ $\beta_{\langle\rangle}$ [4]

λ

$\Lambda_{\langle\rangle}$ $\langle P, Q\rangle\, R$

$\pi_i(\lambda x.P)$

$\Lambda_{\langle\rangle}$

$L_{\langle\rangle}$ $\Lambda_{\langle\rangle}$

$$L_{\langle\rangle} \ni \tau ::= \alpha \mid (\tau \to \tau') \mid (\tau \times \tau') \mid \bot$$

α

$\to$

$\neg\phi \equiv \phi \to \bot$

$\Gamma_{\langle\rangle}$

$\Gamma \in \Gamma_{\langle\rangle}$ $\operatorname{dom}\Gamma$ $|\Gamma|$

$x : \tau$ $\{x : \tau\}$ Γ, Δ

$\Gamma \cup \Delta$ $\operatorname{dom}\Gamma \cap \operatorname{dom}\Delta = \emptyset$

$\vdash \subseteq \Gamma_{\langle\rangle} \times \Lambda_{\langle\rangle} \times L_{\langle\rangle}$ $\Lambda_{\langle\rangle}$

$$\frac{}{\Gamma, x : \tau \vdash x : \tau}$$

$$\frac{\Gamma, x : \tau \vdash P : \sigma}{\Gamma \vdash \lambda x.P : \tau \to \sigma} \to I \qquad \frac{\Gamma \vdash P : \tau \to \sigma \quad \Gamma \vdash Q : \tau}{\Gamma \vdash P\, Q : \sigma} \to E$$

$$\frac{\Gamma \vdash P : \tau \quad \Gamma \vdash Q : \sigma}{\Gamma \vdash \langle P, Q\rangle : \tau \times \sigma} \times I \qquad \frac{\Gamma \vdash P : \tau_1 \times \tau_2}{\Gamma \vdash \pi_i(P) : \tau_i} \times E$$

β

$\Gamma \vdash M : \tau$ $M\!-\!>_\beta N$ $\Gamma \vdash N : \tau$

Λ^I $\Lambda_{\langle\rangle}$

[4] λ

$\Lambda^{I}_{\langle\rangle}$ $\Lambda_{\langle\rangle}$

$\lambda x.P \subseteq M \Rightarrow x \in \mathrm{FV}(P)$
$\langle P, Q\rangle \subseteq M \Rightarrow \mathrm{FV}(P) = \mathrm{FV}(Q)$

$\Lambda^{I}_{\langle\rangle}$ β $\Lambda^{I}_{\langle\rangle}$
$(\lambda x.\pi_1(\langle x, x\,x\rangle))\,\omega$

$$(\lambda x.\pi_1(\langle x, x\,x\rangle))\,\omega -\!>_{\beta_\lambda} \pi_1(\langle \omega, \Omega\rangle) -\!>_{\beta_{\langle\rangle}} \omega$$

$$(\lambda x.\pi_1(\langle x, x\,x\rangle))\,\omega -\!>_{\beta_\lambda} \pi_1(\langle \omega, \Omega\rangle) -\!>_{\beta_\lambda} \pi_1(\langle \omega, \Omega\rangle) -\!>_{\beta_\lambda} \cdots\ .$$

$\beta_{\langle\rangle}$ β_λ $\beta_{\langle\rangle}$
Λ^{I}
Q $(\lambda x.P)\,Q$
Λ^{I} $\beta_{\langle\rangle}$

$$(\lambda x.\pi_1(\langle x, x\,x\rangle))\,\omega -\!>_{\beta} (\lambda x.x)\,\omega -\!>_{\beta} \omega$$

$$x\,x \in \mathrm{NF}_\beta \qquad (x\,x)\{x := \omega\} \in \infty_\beta$$

β_λ $(\lambda x.P)\,Q$ P Q
$P\{x := Q\}$

ι $\Lambda^{[]}_{\langle\rangle}$ $\Lambda_{\langle\rangle}$

$\Lambda^{[]}_{\langle\rangle}$

$\Lambda^{[]}_{\langle\rangle}$ $\Lambda^{[]}_{\langle\rangle}$

$\Lambda_{\langle\rangle}$

$$\Lambda^{[]}_{\langle\rangle} \ni M ::= x \mid \lambda x.M \mid M\,M \mid \langle M, M\rangle \mid \pi_1(M) \mid \pi_2(M) \mid [M \mid M]$$

$M \in \Lambda_{\langle\rangle}$ $M' \in \Lambda^{[]}_{\langle\rangle}$

M'

ι

β

$\iota : \Lambda_{\langle\rangle} \to \Lambda^{[]}_{\langle\rangle}$

$M \in \Lambda_{\langle\rangle}$

$$\begin{aligned}
\iota(x) &\equiv x \\
\iota(\lambda x.P) &\equiv \lambda x.[\iota(P) \mid x] \\
\iota(P\,Q) &\equiv \iota(P)\,\iota(Q) \\
\iota(\langle P, Q\rangle) &\equiv \langle[\iota(P) \mid \iota(Q)], [\iota(Q) \mid \iota(P)]\rangle \\
\iota(\pi_i(P)) &\equiv \pi_i(\iota(P))
\end{aligned}$$

β

$$\begin{array}{rcccc}
\iota((\lambda x.P)\,Q) & \equiv & (\lambda x.[P' \mid x])\,Q' & -->_{\beta_\lambda} & [P'\{x := Q'\} \mid Q'] \\
\iota(\pi_1(\langle P, Q\rangle)) & \equiv & \pi_1(\langle[P' \mid Q'], [Q' \mid P']\rangle) & -->_{\beta_{\langle\rangle}} & [P' \mid Q'] \\
\iota(\pi_2(\langle P, Q\rangle)) & \equiv & \pi_2(\langle[P' \mid Q'], [Q' \mid P']\rangle) & -->_{\beta_{\langle\rangle}} & [Q' \mid P']
\end{array}$$

β $\Lambda_{\langle\rangle}$ M

$$\pi_1((\lambda x.\langle x, x\,x\rangle)\,\omega)\,y$$

$$\begin{array}{rl}
 & \pi_1((\lambda x.\langle x, x\,x\rangle)\,\omega)\,y \\
->_{\beta_\lambda} & \pi_1(\langle \omega, \Omega\rangle)\,y \\
->_{\beta_{\langle\rangle}} & \omega\,y \\
->_{\beta_\lambda} & y\,y\ \ .
\end{array}$$

ι β_λ
β_λ

$$\begin{aligned} & \pi_1((\lambda x.[\langle [x \mid x\, x], [x\, x \mid x]\rangle \mid x])\, \lambda z.[z\, z \mid z])\, y \\ -\!>_{\beta_\lambda} \quad & \pi_1([\langle [Z \mid Z\, Z], [Z\, Z \mid Z]\rangle \mid Z])\, y \end{aligned}$$

$Z \equiv \lambda z.[z\, z \mid z]$ $\beta_{\langle\rangle}$

β_λ $Z\, Z -\!>_{\beta_\lambda} [Z\, Z \mid Z]$

π

κ

$\Lambda^{[]}_{\langle\rangle}$ β
$\Lambda_{\langle\rangle}$ $\Lambda^{[]}_{\langle\rangle}$

$$[P \mid Q]\, R \;\; \pi_\lambda \;\; [P\, R \mid Q]$$
$$\pi_i([P \mid Q]) \;\; \pi_{\langle\rangle} \;\; [\pi_i(P) \mid Q]$$
$$[P \mid Q] \;\; \kappa \;\; P$$

P Q R $\Lambda^{[]}_{\langle\rangle}$ π $\pi = \pi_\lambda \cup \pi_{\langle\rangle}$

$$\begin{aligned} \cdots \to_{\pi_{\langle\rangle}} \quad & [\pi_1(\langle [Z \mid Z\, Z], [Z\, Z \mid Z]\rangle) \mid Z]\, y \\ -\!>_{\beta_{\langle\rangle}} \quad & [[Z \mid Z\, Z] \mid Z]\, y \\ \twoheadrightarrow^2_{\pi_\lambda} \quad & [[Z\, y \mid Z\, Z] \mid Z] \\ -\!>_{\beta_\lambda} \quad & [[[y\, y \mid y] \mid Z\, Z] \mid Z] \end{aligned}$$

β π β

κ

$$\cdots -\!>_\kappa \quad [[y\, y \mid y] \mid Z\, Z] \quad -\!>_\kappa \quad [y\, y \mid y] \quad -\!>_\kappa \quad y\, y \; .$$

$\Lambda^{[]}_{\langle\rangle}$ $\Lambda_{\langle\rangle}$ $M \in \Lambda^{[]}_{\langle\rangle}$
$\Lambda_{\langle\rangle}$ $\beta \cup \pi$

β

Y

$\underline{\bullet}$:

$\Lambda^{[]}_{\langle\rangle} \to \Lambda_{\langle\rangle}$ $M \in \Lambda^{[]}_{\langle\rangle}$

$$
\begin{aligned}
\underline{x} &\equiv \lambda k.x\,k \\
\underline{\lambda x.P} &\equiv \lambda k.k\,\lambda x.\underline{P} \\
\underline{\langle P, Q\rangle} &\equiv \lambda k.k\,\langle \underline{P}, \underline{Q}\rangle \\
\underline{P\,Q} &\equiv \lambda k.\underline{P}\,\lambda m.m\,\underline{Q}\,k \\
\underline{\pi_i(P)} &\equiv \lambda k.\underline{P}\,\lambda m.\pi_i(m)\,k \\
\underline{[P \mid Q]} &\equiv \lambda k.y\,(\underline{P}\,k)\,\underline{Q}
\end{aligned}
$$

$y \in Y$ $\underline{M}$
k m

y
$\underline{P}$ $\underline{Q}$
$[P \mid Q]$ P Q y
P Q
y

ι $\underline{\bullet}$ $\underline{\iota(\bullet)}$
$\Lambda_{\langle\rangle}$ $\Lambda_{\langle\rangle}$ $M \in \Lambda_{\langle\rangle}$ [5]

$$
\begin{aligned}
\underline{\iota(x)} &\equiv \lambda k.x\,k \\
\underline{\iota(\lambda x.P)} &\equiv \lambda k.k\,\lambda x.\lambda l.y\,(\underline{\iota(P)}\,l)\lambda m.x\,m \\
\underline{\iota(\langle P, Q\rangle)} &\equiv \lambda k.k\,\langle \lambda l'.y'\,(\underline{\iota(P)}\,l')\,\underline{\iota(Q)}, \lambda l''.y''\,(\underline{\iota(Q)}\,l'')\,\underline{\iota(P)}\rangle \\
\underline{\iota(P\,Q)} &\equiv \lambda k.\underline{\iota(P)}\,\lambda m.m\,\underline{\iota(Q)}\,k \\
\underline{\iota(\pi_i(P))} &\equiv \lambda k.\underline{\iota(P)}\,\lambda m.\pi_i(m)\,k
\end{aligned}
$$

5 l l' l'' m

y y' y'' k l l' l''

$\underline{\iota(M)} \in \mathrm{WN}_\beta \Rightarrow M \in \mathrm{SN}_\beta$

$\Lambda_{\langle\rangle}$ $\Lambda^{[]}_{\langle\rangle}$

$\Lambda_{\langle\rangle}$

$\Lambda^{[]}_{\langle\rangle}$

$\underline{\iota(M)}$

$[\bullet]$

$[\bullet], [\bullet]' : L_{\langle\rangle} \to L_{\langle\rangle}$

$\tau \in L_{\langle\rangle}$

$$[\tau] \equiv \neg\neg[\tau]'$$

$$\begin{array}{lcl}
[\bot]' & \equiv & \bot \\
[\alpha]' & \equiv & \alpha \\
[\tau \to \sigma]' & \equiv & [\tau] \to [\sigma] \\
[\tau \times \sigma]' & \equiv & [\tau] \times [\sigma]
\end{array}$$

$[\bullet]$ $\Gamma \in \Gamma_{\langle\rangle}$ $[\Gamma] = \{x : [\tau] \mid x : \tau \in \Gamma\}$

$M \in \Lambda_{\langle\rangle}$ $\tau \in L_{\langle\rangle}$ $\Gamma \in \Gamma_{\langle\rangle}$
$\Gamma \vdash M : \tau$

$$\Delta, [\Gamma] \vdash \underline{\iota(M)} : [\tau]$$

$\Delta \in \Gamma_{\langle\rangle}$ $\mathrm{dom}\,\Delta = \mathrm{FV}(\underline{\iota(M)}) \setminus \mathrm{FV}(M)$

$\Gamma \vdash M : \tau$
Δ y
$\square$

$M \in \Lambda_{\langle\rangle}$ $\underline{\iota(M)} \in \Lambda_{\langle\rangle}$

$\underline{\bullet}$

$M - >_\pi N$ $\quad \underline{M} \to_\beta \underline{N}$ $\quad \underline{M} =_\beta \underline{N}$

$[P \mid Q]\, R - >_\pi [P\, R \mid Q]$

$$\underline{[P \mid Q]\, R} \quad \equiv \quad \lambda k.(\lambda l.y\, (\underline{P}\, l)\, \underline{Q})\, \lambda m.m\, \underline{R}\, k$$

$$\underline{[P\, R \mid Q]} \quad \equiv \quad \lambda k.y\, ((\lambda l.\underline{P}\, (\lambda m.m\, \underline{R}\, l))\, k)\, \underline{Q}.$$

$$\lambda k.y\, (\underline{P}\, (\lambda m.m\, \underline{R}\, k))\, \underline{Q}.$$

$\underline{[P \mid Q]\, R} =_\beta \underline{[P\, R \mid Q]}$ $\quad \underline{[P \mid Q]\, R} - >_\beta \underline{[P\, R \mid Q]}$

$\underline{\underline{\bullet}}$ $\quad M - >_{\beta\pi} N$

$\underline{\underline{M}} \twoheadrightarrow_\beta \underline{\underline{N}}$

$\underline{M} \twoheadrightarrow_\beta \underline{\underline{M}}$ $\quad \underline{\underline{\bullet}}$

$\underline{\underline{\bullet}}$ $\quad \bullet : \bullet$

Y

$$\underline{\underline{\bullet}} : \Lambda^{[]}_{\langle\rangle} \to \Lambda_{\langle\rangle} \qquad \bullet : \bullet : \Lambda^{[]}_{\langle\rangle} \times \Lambda_{\langle\rangle} \longrightarrow \Lambda_{\langle\rangle}$$

$M \in \Lambda^{[]}_{\langle\rangle}$

$$\underline{\underline{M}} \qquad \equiv \lambda m.(M : m)$$

$$\begin{array}{lcl}
x : K & \equiv & x\, K \\
(\lambda x.P) : K & \equiv & K\, (\lambda x.\underline{\underline{P}}) \\
\langle P, Q\rangle : K & \equiv & K\, \langle \underline{\underline{P}}, \underline{\underline{Q}}\rangle \\
(P\, Q) : K & \equiv & P : (\lambda m.m\, \underline{\underline{Q}}\, K) \\
\pi_i(P) : K & \equiv & P : (\lambda m.\pi_i(m)\, K) \\
[P \mid Q] : K & \equiv & y\, (P : K)\, \underline{\underline{Q}}
\end{array}$$

$y \in Y$ $\quad y$ $\quad \underline{\underline{M}}$

m

$\underline{\bullet}$

$$\underline{P}\, Q \equiv (\lambda k.P')\, Q$$

$P'\{k := Q\}$

k $\quad \bullet : \bullet$

K $\quad k$

$\Lambda_{\langle\rangle}$

$\underline{\iota(\bullet)}$

$\Lambda_{\langle\rangle}$ $\underline{\iota(M)} \in \mathrm{WN}_\beta \Rightarrow \underline{\iota(M)} \in \mathrm{SN}_\beta$

$\Lambda_{\langle\rangle}$

$\Lambda^I_{\langle\rangle}$ β_λ

$\beta_{\langle\rangle}$

P Q $\beta_{\langle\rangle}$

Y

Y

Y θ Y $y \in Y$

$y\theta \equiv y$

Y Y

Y

Y

Y $\langle P, Q\rangle \in \Lambda_{\langle\rangle}$ Y

Y θ

$$P\theta \in \infty_\beta \Longleftrightarrow Q\theta \in \infty_\beta\ .$$

Y

$\Lambda_{\langle\rangle}$

Y

$\Lambda_{\langle\rangle}$

β

$\mathcal{E}_l$ $\Lambda_{\langle\rangle}$

$$E ::= [\,] \mid E\,P \mid \pi_i(E)$$

P $\Lambda_{\langle\rangle}$

$\Lambda_{\langle\rangle}$

$M \in \Lambda_{\langle\rangle}$
$E \in \mathcal{E}_l$
$R \in \Lambda_{\langle\rangle}$ $E[R] \equiv M$

$R \equiv x$
$R \equiv \lambda x.P$ $E \equiv [\,]$
$R \equiv \langle P, Q\rangle$ $E \equiv [\,]$
$R \equiv (\lambda x.P)\,Q$
$R \equiv \pi_i(\langle P, Q\rangle)$

P Q $\Lambda_{\langle\rangle}$

$M \in \Lambda_{\langle\rangle}$
$\mathrm{rel}(M)$ M

$$\begin{aligned}
\mathrm{rel}(x) &= \emptyset \\
\mathrm{rel}(E[x]\ P) &= \mathrm{rel}(E[x]) \cup \mathrm{rel}(P) \\
\mathrm{rel}(\pi_i(E[x])) &= \mathrm{rel}(E[x]) \\
\mathrm{rel}(\lambda x.P) &= \mathrm{rel}(P) \\
\mathrm{rel}(\langle P, Q\rangle) &= \mathrm{rel}(P) \cup \mathrm{rel}(Q) \\
\mathrm{rel}(E[(\lambda x.P)\,Q]) &= \{\langle S, T\rangle \mid \langle S, T\rangle \subseteq E[(\lambda x.P)\,Q]\} \\
\mathrm{rel}(E[\pi_i(\langle P, Q\rangle)]) &= \{\langle S, T\rangle \mid \langle S, T\rangle \subseteq E[\pi_i(\langle P, Q\rangle)]\}
\end{aligned}$$

M M

$R \equiv x$

Y $M \in \Lambda_{\langle\rangle}$ Y
Y

Y Y
Y
$\Lambda_{\langle\rangle}$

$M \in \Lambda_{\langle\rangle}$
F_l M

$$
\begin{aligned}
F_l(x) &\equiv x \\
F_l(E[x]\ P) &\equiv \begin{cases} F_l(E[x])\ P & E[x] \notin \mathrm{NF}_\beta \\ E[x]\ F_l(P) & \end{cases} \\
F_l(\pi_i(E[x])) &\equiv \pi_i(F_l(E[x])) \\
F_l(\lambda x.P) &\equiv \lambda x.F_l(P) \\
F_l(\langle P, Q\rangle) &\equiv \begin{cases} \langle F_l(P), Q\rangle & P \notin \mathrm{NF}_\beta \\ \langle P, F_l(Q)\rangle & \end{cases} \\
F_l(E[(\lambda x.P)\ Q]) &\equiv E[P\{x := Q\}] \\
F_l(E[\pi_i(\langle P_1, P_2\rangle)]) &\equiv E[P_i]
\end{aligned}
$$

F_l
M $F_l(M)$
F_l

λ
[6]

F_l

$\Lambda^I_{\langle\rangle}$
F_l

6 F_l $\langle P, Q\rangle$
Q $P \notin \mathrm{WN}_\beta$
F_l
F_l
F_l

M $\Lambda^I_{\langle\rangle}$ N $\Lambda_{\langle\rangle}$ $M->_\beta$
N N $\Lambda^I_{\langle\rangle}$

$M->_\beta N$ □

$M \in \Lambda_{\langle\rangle}$ M Y $F_l(M)$
Y

M Y $\Lambda_{\langle\rangle}$
M $F_l(M)$ Y

$M \equiv E[x]$
$E \equiv []$ $\mathrm{rel}(F_l(x)) = \mathrm{rel}(x) \equiv \emptyset$
$E \equiv E'P$
$E \equiv \pi_i(E')$
$M \equiv \langle P, Q\rangle$ $P \notin \mathrm{NF}_\beta$ $F_l(M) \equiv \langle F_l(P), Q\rangle$
$\mathrm{rel}(F_l(M)) = \mathrm{rel}(F_l(P)) \cup \mathrm{rel}(Q)$ $\mathrm{rel}(M) = \mathrm{rel}(P) \cup \mathrm{rel}(Q)$
$\mathrm{rel}(Q)$ Y M Y
$F_l(P)$ Y $\mathrm{rel}(F_l(M))$ Y
$F_l(M)$ Y
$P \in \mathrm{NF}_\beta$ $F_l(M) \equiv \langle P, F_l(Q)\rangle$
$F_l(M)$ Y
$M \equiv \lambda x.P$
$M \equiv E[(\lambda x.P)\, Q]$ M Y Y
$E[(\lambda x.P)\, Q]$
M Y
$\langle S, T\rangle \subseteq F_l(M) \equiv E[P\{x := Q\}]$

$\langle S, T\rangle \subseteq Q$ $\langle S, T\rangle \subseteq E$ $\langle S, T\rangle \subseteq M$
Y
$\langle S, T\rangle \subseteq P\{x := Q\}$ $\langle S, T\rangle \not\subseteq Q$
$\langle S', T'\rangle \subseteq P$ $\langle S', T'\rangle\{x := Q\} \equiv \langle S, T\rangle$
θ Y
$x \notin Y$ $\{x := Q\}\theta$ Y
$\langle S', T'\rangle$ Y

$$\infty_\beta \ni S\theta \equiv S'\{x := Q\}\theta \iff T'\{x := Q\}\theta = T\theta \in \infty_\beta$$

$\langle S, T\rangle$ Y
$E[P\{x := Q\}]$ Y
$E[P\{x := Q\}]$ Y
$M \equiv E[\pi_i(\langle P_1, P_2\rangle)]$ M Y
Y $E[\pi_i(\langle P_1, P_2\rangle)]$
M Y

$\langle S, T\rangle \subseteq F_l(M) \equiv E[P_i]$ $\langle S, T\rangle \subseteq P_i$
$\langle S, T\rangle \subseteq E$ $\langle S, T\rangle \subseteq M$ Y
$E[\pi_i(\langle P_1, P_2\rangle)]$
Y Y

Y
F_l $\square$

$\langle P, Q\rangle$ [7] F_l

$M \in \Lambda^I_{\langle\rangle}$ Y $M \in \infty_\beta$
$F_l(M) \in \infty_\beta$

M
M $F_l(M)$

$M \equiv E[x]$
$E \equiv [\,]$ $x \in \mathrm{NF}_\beta$ $x \in \infty_\beta$
$E \equiv E'\, P$ $E'[x] \twoheadrightarrow_\beta \lambda x.S$
$M \in \infty_\beta$
$E'[x] \in \infty_\beta$ $F_l(M) \equiv F_l(E'[x])\, P$
$F_l(E'[x]) \in \infty_\beta$ $F_l(M) \in \infty_\beta$
$P \in \infty_\beta$ $E'[x] \notin \mathrm{NF}_\beta$ $F_l(M) \equiv F_l(E'[x])\, P$
$F_l(M) \in \infty_\beta$
$E'[x] \in \mathrm{NF}_\beta$ $F_l(M) \equiv E'[x]\, F_l(P)$
$F_l(P) \in \infty_\beta$ $F_l(M) \in \infty_\beta$
$E \equiv \pi_i(E')$ $E'[x] \twoheadrightarrow_\beta \langle S_1, S_2\rangle$
$M \in \infty_\beta$ $E'[x] \in \infty_\beta$
$F_l(E'[x]) \in \infty_\beta$ $F_l(M) \equiv \pi_i(F_l(E'[x])) \in \infty_\beta$
$M \equiv \lambda x.P$ $\lambda x.P \in \infty_\beta$ $P \in \infty_\beta$
$F_l(P) \in \infty_\beta$ $F_l(M) \equiv \lambda x.F_l(P) \in \infty_\beta$
$M \equiv \langle P, Q\rangle$
$M \equiv E[(\lambda x.P)\, Q]$ $F_l(M) \equiv E[P\{x := Q\}]$
$E[(\lambda x.P)\, Q] \in \infty_\beta$
$Q \in \infty_\beta$ $M \in \Lambda^I_{\langle\rangle}$ $x \in \mathrm{FV}(P)$ $Q \subseteq E[P\{x := Q\}]$
$F_l(M) \in \infty_\beta$

---
[7] Y β_λ
Y $\Lambda^I_{\langle\rangle}$
K

$\lambda x.P \in \infty_\beta$ $P \in \infty_\beta$ $P\{x := Q\} \in \infty_\beta$
$F_l(M) \in \infty_\beta$
$E \in \infty_\beta$ $F_l(M) \equiv E[P\{x := Q\}] \in \infty_\beta$
$E[(\lambda x.P)\, Q] \twoheadrightarrow_\beta E'[(\lambda x.P')\, Q'] -\!>_\beta E'[P'\{x := Q'\}] \in \infty_\beta$
$E \twoheadrightarrow_\beta E'$ $P \twoheadrightarrow_\beta P'$ $Q \twoheadrightarrow_\beta Q'$

$$\begin{aligned} F_l(M) \equiv E[P\{x := Q\}] &\twoheadrightarrow_\beta E'[P\{x := Q\}] \\ &\twoheadrightarrow_\beta E'[P'\{x := Q\}] \\ &\twoheadrightarrow_\beta E'[P'\{x := Q'\}] \in \infty_\beta \end{aligned}$$

$M \equiv E[\pi_i(\langle P_1, P_2\rangle)]$ $F_l(M) \equiv E[P_i]$
$M \in \infty_\beta$
$P_i \in \infty_\beta$ $E \in \infty_\beta$ $F_l(M) \in \infty_\beta$
$P_j \in \infty_\beta$ $j \neq i$ $\langle P_1, P_2\rangle$ M
$\emptyset$ Y

$$P_j\emptyset \in \infty_\beta \iff P_i\emptyset \in \infty_\beta \quad P_i \in \infty_\beta$$

$E[\pi_i(\langle P_1, P_2\rangle)] \twoheadrightarrow_\beta E'[\pi_i(\langle P_1', P_2'\rangle)] -\!>_\beta E'[P_i'] \in \infty_\beta$ $E \twoheadrightarrow_\beta$
E' $P_j \twoheadrightarrow_\beta P_j'$ $j = 1, 2$

$$\begin{aligned} E[P_i] &\twoheadrightarrow_\beta E'[P_i] \\ &\twoheadrightarrow_\beta E'[P_i'] \in \infty_\beta \end{aligned}$$

F_l $\square$

M Y $M \in \Lambda^I_{\langle\rangle}$
Y

$$M \in \Lambda^I_{\langle\rangle}$$

$M \in \Lambda^I_{\langle\rangle}$ Y $M \in \mathrm{UN}_\beta$

$M \in \mathrm{WN}_\beta \Rightarrow M \in \mathrm{SN}_\beta$ $M \in \mathrm{WN}_\beta$
F_l

$$M -\!>_\beta F_l(M) -\!>_\beta F_l^2(M) -\!>_\beta \cdots -\!>_\beta F_l^n(M) \in \mathrm{NF}_\beta$$

$M \in \mathrm{SN}_\beta$ $M \in \infty_\beta$
M $F_l^i(M)$
n

$F_l^i(M)$ Y $\Lambda_{\langle\rangle}^I$ $F_l^i(M) \in \infty_\beta$
$F_l^n(M) \in \infty_\beta$ $F_l^n(M) \in \mathrm{NF}_\beta$
$M \notin \infty_\beta$ $M \in \mathrm{UN}_\beta$ $\square$

$$\underline{\iota(M)} \in \mathrm{WN}_\beta \Longrightarrow \underline{\iota(M)} \in \mathrm{SN}_\beta$$

$\underline{\iota(\bullet)}$
$\underline{\iota(M)}$
$M \in \Lambda_{\langle\rangle}$
$\underline{\iota(\bullet)}$ $\Lambda_{\langle\rangle}$ Y $\Lambda_{\langle\rangle}^I$

$M \in \Lambda_{\langle\rangle}$ $\underline{\iota(M)} \in \Lambda_{\langle\rangle}^I$

M $\square$

$M \in \Lambda_{\langle\rangle}$ $\underline{\iota(M)}$ Y
Y $\underline{\bullet}$

M $\underline{\iota(M)}$
$\underline{\iota(M)}$
$\langle S, T\rangle \subseteq \underline{\iota(M)}$ Y Y $\underline{\bullet}$
M $\langle S, T\rangle \subseteq \underline{\iota(M)}$
Y

$M \equiv x$ $\underline{\iota(M)} \equiv \lambda k.x\, k$ $\langle S_1, S_2\rangle \subseteq \underline{\iota(M)}$
$M \equiv P\, Q$ $M \equiv \lambda x.P$ $M \equiv \pi_i(P)$

$M \equiv P\, Q$ $\underline{\iota(M)} \equiv \lambda k.\underline{\iota(P)}\, \lambda m.m\, \underline{\iota(Q)}\, k$
$\langle S, T\rangle \subseteq \underline{\iota(M)}$ $\langle S, T\rangle \subseteq \underline{\iota(P)}$ $\langle S, T\rangle \subseteq \underline{\iota(Q)}$
$\langle S, T\rangle$ Y

$M \equiv \langle P_1, P_2\rangle$ $\underline{\iota(M)} \equiv \lambda k.k\, \langle R_1, R_2\rangle$
$R_i \equiv \lambda l^{(i)}.y^{(i)}\, (\underline{\iota(P_i)}\, l^{(i)})\, \underline{\iota(P_j)}$ $j = i + 1 \bmod 2$
$\langle S, T\rangle \subseteq \underline{\iota(M)}$
$\langle S, T\rangle \subseteq \underline{\iota(P_i)}$ $i = 1, 2$ $\langle S, T\rangle$ Y

$\langle S, T\rangle \equiv \langle R_1, R_2\rangle$ $R_1\theta \in \infty_\beta \Longleftrightarrow R_2\theta \in \infty_\beta$
Y θ Y θ
$R_i\theta \equiv \lambda l^{(i)}.y^{(i)}\, (\underline{\iota(P_i)}\theta\, l^{(i)})\, \underline{\iota(P_j)}\theta$ $j = i + 1 \bmod 2$
$R_1\theta \in \infty_\beta$ $R_2\theta \in \infty_\beta$

$\underline{\iota(P_i)}\theta \in \infty_\beta$ $i = 1, 2$ $R_2\theta \in \infty_\beta$
$\underline{\iota(P_1)}\theta\, l^{(1)} \in \infty_\beta$ $\underline{\iota(P_1)} \equiv \lambda k'.P'$
P' $\underline{\iota(P_1)}\theta \equiv \lambda k'.P'\theta$ $(\underline{\iota(P_1)}\theta)\, l^{(1)}$
$P'\theta$ $P'\theta \in \infty_\beta$ $\underline{\iota(P)}\theta \in$
∞_β $R_2\theta \in \infty_\beta$
$R_1\theta \in \infty_\beta$ $R_2\theta \in \infty_\beta$ $R_2\theta \in \infty_\beta$
$R_1\theta \in \infty_\beta$ R_1 R_2

$\underline{\iota(M)}$ Y □

$M \in \Lambda_{\langle\rangle}$ $\underline{\iota(M)} \in \mathrm{UN}_\beta$

$$\underline{\iota(M)} \in \mathrm{SN}_\beta \Longrightarrow \iota(M) \in \mathrm{SN}_{\beta\pi}$$

$\Lambda_{\langle\rangle}$

$$\underline{\iota(M)} \in \mathrm{SN}_\beta \Longrightarrow \iota(M) \in \mathrm{SN}_{\beta\pi}$$

$$\underline{M} \in \mathrm{SN}_\beta \Longrightarrow M \in \mathrm{SN}_{\beta\pi} \ .$$

β $\underline{M}$
$\beta\pi$ $\beta\pi$
β $\underline{M}$

$\underline{\bullet}$ $\underline{\underline{\bullet}}$

$\beta\pi$

$\Lambda^{[]}_{\langle\rangle}$ $\Lambda_{\langle\rangle}$ $\beta\pi$
β

$\chi : \Lambda^{[]}_{\langle\rangle} \to \Lambda_{\langle\rangle}$ $\beta\pi$

$$L{-}>_\beta K \Longrightarrow \chi(L) \twoheadrightarrow^+_\beta \chi(K)$$
$$L{-}>_\pi K \Longrightarrow \chi(L) \twoheadrightarrow_\beta \chi(K)$$

$L, K \in \Lambda^{[]}_{\langle\rangle}$

β

$\psi, \chi \,:\, \Lambda^{[]}_{\langle\rangle} \;\to\; \Lambda_{\langle\rangle}$ χ

$\beta\pi$ $\psi(M) \twoheadrightarrow_\beta \chi(M)$ $M \in \Lambda^{[]}_{\langle\rangle}$

$$\psi(M) \in \mathrm{SN}_\beta \Longrightarrow M \in \mathrm{SN}_{\beta\pi} \;.$$

$\underline{\bullet}$

$M- >_{\beta\pi} N$

$\underline{M} =_\beta \underline{N}$ $\underline{M} \twoheadrightarrow_\beta \underline{N}$

$\underline{\underline{\bullet}}$ $\underline{\underline{M}} \twoheadrightarrow_\beta \underline{\underline{N}}$ $M- >_{\beta\pi} N$ $\underline{M} \twoheadrightarrow_\beta \underline{\underline{M}}$

π

$M \in \Lambda^{[]}_{\langle\rangle}$ $M \in \mathrm{SN}_\pi$

$$w : \Lambda^{[]}_{\langle\rangle} \to \mathbb{N}$$

$$\begin{aligned} w(x) &= 1 \\ w(\lambda x.P) &= w(P) \\ w(P\,Q) &= 2w(P) + w(Q) \\ w(\pi_i(P)) &= 2w(P) \\ w(\langle P, Q\rangle) &= w(P) + w(Q) \\ w([P \mid Q]) &= w(P) + w(Q) \end{aligned}$$

$M \to_\pi N \Longrightarrow w(M) > w(N)$ $w(M) \in \mathbb{N}$

M π

π M $\square$

M

$\beta\pi$

π β β

χ $\beta\pi$

β $\chi(M)$ $\chi(M) \in \infty_\beta$
$\psi(M) \twoheadrightarrow_\beta \chi(M) \in \infty_\beta$ $\psi(M) \in \mathrm{SN}_\beta$ $M \in \mathrm{SN}_{\beta\pi}$ $\square$

$\beta\pi$

$\psi = \underline{\bullet}$ $\chi = \underline{\underline{\bullet}}$

M $\Lambda^{[]}_{\langle\rangle}$ K L $\Lambda_{\langle\rangle}$ k
$k \notin \mathrm{FV}(M)$

$$(M\!:\!K)\{k := L\} \equiv M\!:\!(K\{k := L\}) \;.$$

$P, Q \in \Lambda^{[]}_{\langle\rangle}$

$$\underline{\underline{P}}\, Q \equiv (\lambda k.P\!:\!k)\, Q{-}>_\beta (P\!:\!k)\{k := Q\} \equiv P\!:\!Q$$

k

M $\square$

M $\Lambda^{[]}_{\langle\rangle}$ K L $\Lambda_{\langle\rangle}$

$$K \twoheadrightarrow^+_\beta L \Longrightarrow M\!:\!K \twoheadrightarrow^+_\beta M\!:\!L \;.$$

M $\square$

M N $\Lambda^{[]}_{\langle\rangle}$ K $\Lambda_{\langle\rangle}$

$$(M\!:\!K)\{x := \underline{\underline{N}}\} \twoheadrightarrow_\beta (M\{x := N\})\!:\!(K\{x := \underline{\underline{N}}\}) \;.$$

M

$$\underline{\underline{M}}\{x := \underline{\underline{N}}\} \equiv \lambda k.(M\!:\!k)\{x := \underline{\underline{N}}\} \twoheadrightarrow_\beta \lambda k.(M\{x := N\}\!:\!k) \equiv \underline{\underline{M\{x := N\}}} \;.$$

$\square$

$M, N \in \Lambda^{[]}_{\langle\rangle}$ $K \in \Lambda_{\langle\rangle}$

$\underline{M} \twoheadrightarrow_\beta \underline{\underline{M}}$
$M \to_\beta N \Longrightarrow M\!:\!K \twoheadrightarrow^+_\beta N\!:\!K$

$M \to_\pi N \Longrightarrow M\!:\!K \equiv N\!:\!K$

M

$$\underline{\underline{P}}\, Q \equiv (\lambda k.P\!:\!k)\, Q -\!>_\beta (P\!:\!k)\{k := Q\} \equiv P\!:\!Q$$

β

π

$\square$

$M \in \Lambda_{\langle\rangle}$

$$\underline{\iota(M)} \in \mathrm{SN}_\beta \Longrightarrow \iota(M) \in \mathrm{SN}_{\beta\pi}$$

$\underline{\underline{M}} \equiv \lambda m.M\!:\!m$

$\psi = \underline{\bullet} \qquad \chi = \underline{\underline{\bullet}}$ $\square$

$\iota(M) \in \mathrm{SN}_{\beta\pi} \Longrightarrow M \in \mathrm{SN}_\beta$

$\Lambda_{\langle\rangle}$

$$\iota(M) \in \mathrm{SN}_{\beta\pi} \Longrightarrow M \in \mathrm{SN}_\beta \ .$$

$$\iota(M) \in \mathrm{SN}_{\beta\pi} \Longrightarrow \iota(M) \in \mathrm{SN}_{\beta\pi\kappa} \Longrightarrow \iota(M) \in \mathrm{SN}_{\beta\kappa} \Longrightarrow M \in \mathrm{SN}_\beta \ .$$

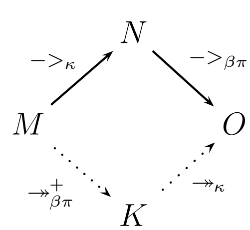
κ

$\beta\kappa$ $\beta\pi\kappa$

$\iota(M) \twoheadrightarrow_\kappa M$

$M, N, O \in \Lambda^{[]}_{\langle\rangle}$ $M -\!>_\kappa N -\!>_{\beta\pi} O$

$K \in \Lambda^{[]}_{\langle\rangle}$ $M \twoheadrightarrow^+_{\beta\pi} K \twoheadrightarrow_\kappa O$



$M -\!>_\kappa N$

$M -\!>_\kappa N \to O$ $\square$

M $\Lambda_{\langle\rangle}^{[]}$ $M \in \mathrm{SN}_{\beta\pi}$ $M \in \mathrm{SN}_{\beta\pi\kappa}$

$M \in \infty_{\beta\pi\kappa}$

$M \in \infty_{\beta\pi}$ n $n \geq 0$

n

$\beta\pi$

$$M -\!> _{\beta\pi} M_1 -\!> _{\beta\pi} \ldots -\!> _{\beta\pi} M_{n-1} -\!> _{\beta\pi} M_n -\!> _{\beta\pi\kappa} M_{n+1} -\!> _{\beta\pi\kappa} \ldots \ .$$

$n = 0$ $M \in \infty_{\beta\pi\kappa}$

$n > 0$

$n - 1$ $\beta\mathrm{p}\pi$

$$M -\!> _{\beta\pi} M_1 -\!> _{\beta\pi} \ldots -\!> _{\beta\pi} M_{n-1} -\!> _{\beta\pi\kappa} M_n -\!> _{\beta\pi\kappa} \cdots \ .$$

κ

$$M_{n-1} -\!> _{\kappa} M_n -\!> _{\kappa} \cdots$$

κ $\mathbb{N}$

$k \geq n - 1$ $M_k -\!> _{\beta\pi} M_{k+1}$ $k - (n-1)$

$$M -\!> _{\beta\pi} M_1 -\!> _{\beta\pi} \cdots -\!> _{\beta\pi} M_{n-1} \twoheadrightarrow^{+}_{\beta\pi} M_n -\!> _{\beta\pi\kappa} M_{n+1} -\!> _{\beta\pi\kappa} \cdots \ .$$

n $\beta\pi$

n $\beta_{\langle\rangle}$ n M

M $\beta\pi$

M $\beta\pi$

$\beta\pi\kappa$ $\square$

M $\Lambda_{\langle\rangle}$ $\iota(M) \twoheadrightarrow_{\kappa} M$

M $\square$

M $\Lambda_{\langle\rangle}$ $\iota(M) \in \mathrm{SN}_{\beta\kappa}$ $M \in \mathrm{SN}_{\beta}$

$M \in \infty_{\beta}$

M

$$M \rightarrow_{\beta} M_1 \rightarrow_{\beta} M_2 \rightarrow_{\beta} \ldots \ .$$

$$\iota(M) \twoheadrightarrow_{\kappa} M \to_{\beta} M_1 \to_{\beta} M_2 \to_{\beta} \ldots$$

$\beta\kappa$ $\iota(M)$ □

$\iota(M) \in \mathrm{SN}_{\beta\pi}$
$M \in \mathrm{SN}_{\beta}$

M $\Lambda_{\langle\rangle}$ $\iota(M) \in \mathrm{SN}_{\beta\pi}$ $M \in \mathrm{SN}_{\beta}$

$\iota(M) \in \mathrm{SN}_{\beta\pi\kappa}$ $\iota(M) \in \mathrm{SN}_{\beta\kappa}$
$M \in \mathrm{SN}_{\beta}$ □

$M \in \Lambda_{\langle\rangle}$ $\underline{\iota(M)} \in \mathrm{WN}_{\beta}$
$M \in \mathrm{SN}_{\beta}$

$\underline{\iota(M)} \in \mathrm{SN}_{\beta}$
$\iota(M) \in \mathrm{SN}_{\beta\pi}$ $M \in$
SN_{β} □

$\Lambda_{\langle\rangle}$

$\Lambda_{\langle\rangle}$

$\Lambda_{\langle\rangle}$

F_l

F_l

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